Roll No. :

B000311(014)

B.Tech. (Third Semester) Examination April-May 2021

(AICTE Scheme)

(All Branches)

MATHEMATICS-III

Time Allowed: Three hours

Maximum Marks: 100

Minimum Pass Marks: 35

Note: Attempt all questions. Part (a) of each unit is compulsory and solve any two parts from (b), (c) and (d) of each questions. Area under the normal curve table is allowed.

Unit-I

1. (a) Evaluate:

 $\int_0^\infty t e^{-2t} \cos t \ dt$

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- (b) Find the Laplace transform of
 - (i) $\frac{\cos at \cos bt}{t}$
 - (ii) $\left(\sqrt{t} \frac{1}{\sqrt{t}}\right)^3$
- (c) Find the inverse Laplace transform of
 - (i) $\frac{2 s^2 6 s + 5}{s^3 6 s^2 + 11 s 6}$
 - (ii) $\tan^{-1}\frac{2}{s^2}$
- (d) Using Laplace transform, solve:

$$y''' - 3y'' + 3y' - y = t^2 e^t$$

given y(0) = 1, y'(0) = 0, y''(0) = -2.

Unit-II

2. (a) Form the partial differential equation from

$$f\left(x^2 + y^2, z - xy\right) = 0$$

(b) Solve:

$$(x^2 - y^2 - z^2) p + 2 xyq = 2 xz$$

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(c) Solve: these are delided a fellow continued the Reference, and 8

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- $(D^2 + DD' 2D'^2)z = (y-1)e^x$
- (d) Using the method of separation of variables solve

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

given that $u(0, y) u = 3 e^{-y} - e^{-5y}$.

BALL DOUGLEROR IL SELDS Unit-III. HOUR SECTION OF THE

- 3. (a) If a random variable has a Poisson distribution such that P(1) = P(2), find
 - (i) mean of the distribution.
 - P(4) supplies P(4)
 - (b) The probability density p(x) of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|}, -\infty < x < \infty$$

Prove that $y_0 = \frac{1}{2}$. Find the mean and variance of the distribution.

- (c) Out of 800 families with 4 children each, how many families would be expected to have
 - (i) 2 boys and 2 girls
 - (ii) at least one by
 - (iii) no girl and
 - (iv) at most two girls

Assume equal probabilities for boys and girls.

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- (d) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S. D. of 60 hours. Estimate the number of bulbs likely to burn for:
 - (i) more than 2150 hours
 - (ii) less than 1950 hours and
 - (iii) more than 1920 hours and less than 2160 hours 8

Unit-IV

4. (a) Find the missing values in the following table:

| x | • | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|----|----|----|----|----|
| у | 2 | 6 | 10 | | 17 | _ | 31 |

[5]

- (b) Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$. Find $\sin 52^\circ$ using Newton's forward interpolation.
- (c) Given the following table, find f(35) by using Stirling's & Bessel's formula.

| x | 20 | 30 | 40 | 50 |
|----------|-----|-----|-----|-----|
| y = f(x) | 512 | 439 | 346 | 243 |

(b) Using Newton's divided difference formula, evaluate f(9) and f(15) from the following table:

| x | 4 | 5 | 7 | 10 | 11 | 13 |
|------|----|-----|-----|-----|------|------|
| f(x) | 48 | 100 | 294 | 900 | 1210 | 2028 |

Unit-V

- 5. (a) If y' = x y, y(0) = 1 then find the value of $y^{(1)}(1)$ by Picard's method.
 - (b) Use Taylor's series method to obain approximate value of y at x = 0.1 for the differential equation, $\frac{dy}{dx} = 2y + 3e^{x} \quad y(0) = 0$

Compare the numerical solution obtained with the exact solution.

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(c) (d) Using modified Euler's method, solve for y at x = 0.1 and 0.2 from

$$\frac{dy}{dx} = y^2 + x^2, \ y(0) = 1.$$

(d) Using Runge-Kutta method of forth order, solve:

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with
$$y(0) = 1'$$
 at $x = 0.2$ and $x = 0.4$.